

On fixed point theorem in Intuitionistic S_b –Menger Spaces

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Abstract: In the present paper, we investigate various classes of mappings, including compatibility of type (E), S-compatible mappings of type (E), and weakly subsequentially continuous mappings in the framework of Intuitionistic S_b -Menger spaces. By employing these newly defined concepts, we establish several common fixed-point theorems under appropriate conditions. The obtained results significantly extend and generalize a number of existing fixed-point theorems in the literature.

Keywords: Fixed Point, Intuitionistic Menger space, Intuitionistic S_b -Menger space Compatible Maps of type(E), Weakly subsequential continuous maps

MSC: 47H10, 54H25.

1. Introduction:

In the history of mathematical analysis, several generalizations of metric spaces have been introduced to overcome limitations of classical metric structures and to develop suitable frameworks for fixed point theory. The concept of 2-metric space was introduced by Gähler [8,9], but its lack of continuity motivated further generalizations. Dhage [7] introduced D-metric spaces, which were later shown to have limitations by Mustafa and Sims [16,17], leading to the development of more consistent structures such as G-metric spaces. Sedghi et al. [25,26] introduced S-metric spaces, which have been widely studied in nonlinear analysis. Probabilistic metric spaces were initiated by Menger [15] and further developed by Schweizer and Sklar [23,24], with significant contributions by Sehgal and Bharucha-Reid [27] and Stojaković [32–34]. Intuitionistic and fuzzy extensions were developed to handle uncertainty, including intuitionistic Menger spaces studied by Kutukcu et al. [12], Pant et al. [18], Jain et al. [11], and Leila and Aliouche [14]. Further developments include cone metric spaces, fuzzy metric type spaces, and other generalizations such as S-Menger and S_b –Menger spaces, studied in [22,13]. The concept of b -metric spaces was introduced by Bakhtin and extended to S_b –structures for broader applications. In fixed point theory, various types of mappings such as compatible mappings, weakly compatible mappings, and sub-compatible mappings were introduced by Jungck and further studied by Bouhadjera and Thobie [3]. Singh and Singh [29,30] introduced compatible mappings of type (E), while Beloul [2] studied weakly subsequential continuous mappings. Property (E.A.) and related concepts were developed in [6,19], and weak contractive conditions were studied in [20].

Control functions and generalized contractions in Menger spaces were investigated by Chaudhary [4] and Chaudhary et al. [5]. Sharma [28] and Rashwan–Hedar [21] extended fixed-point results in intuitionistic and Menger-type spaces. Many recent works focus on intuitionistic S-Menger and S_b –Menger spaces with applications to differential equations and nonlinear systems [13,22].

Motivated by these developments, this paper studies fixed point results for various types of mappings, including compatible mappings of type (E) and weakly subsequential continuous mappings, in intuitionistic S_b –Menger spaces, and establishes new common fixed-point theorems.

2.Preliminaries:

We now recall the following definitions that will be used in the sequel.

Definition 2.1.[23] A mapping $*$: $X^3 \rightarrow X$ ($X = [0,1]$) is said to be CTN if it verifies the below properties:

- (i) $*(a, 1, 1) = a, *(0, 0, 0) = 0$;
- (ii) $*(a, b, c) = *(a, c, b) = *(b, c, a)$;
- (iii) $*$ is continuous;
- (iv) $*(a_1, b_1, c_1) \geq *(a_2, b_2, c_2)$ for $a_1 \geq a_2, b_1 \geq b_2, c_1 \geq c_2$.

$a * b * c = abc$ and $a * b * c = \min\{a, b, c\}$ are called product CTN and minimum CTN respectively.

Definition 2.2. [13] A mapping $*$: $X^3 \rightarrow X$ ($X = [0,1]$) is said to be CTCN if it verifies the below properties:

- (i) $\diamond(a, 1, 1) = a, \diamond(0, 0, 0) = 0$;
- (ii) $\diamond(a, b, c) = \diamond(a, c, b) = \diamond(b, c, a)$;
- (iii) $\diamond$ is continuous;
- (iv) $\diamond(a_1, b_1, c_1) \geq \diamond(a_2, b_2, c_2)$ for $a_1 \geq a_2, b_1 \geq b_2, c_1 \geq c_2$.

Definition 2.3.[13] A distance distribution function is a function $\mathcal{P}: \mathbb{R} \rightarrow \mathbb{R}^+$ which is left continuous on $\mathbb{R}$, non-decreasing and $\inf_{t \in \mathbb{R}} \mathcal{P}(t) = 0, \sup_{t \in \mathbb{R}} \mathcal{P}(t) = 1$.

we shall be denoted by D the family of all distance distribution functions and by H a special distance distribution function in D given by

$$H(t) = \begin{cases} 1, & \text{if } t > 0 \\ 0, & \text{if } t \leq 0 \end{cases}$$

Definition 2.4.[13] A non-distance distribution function is function $\mathcal{Q}: \mathbb{R} \rightarrow \mathbb{R}^+$ which is left continuous on $\mathbb{R}$, non-increasing and $\inf_{t \in \mathbb{R}} \mathcal{Q}(t) = 1, \sup_{t \in \mathbb{R}} \mathcal{Q}(t) = 0$.

We shall denote by E the family of all non-distance distribution functions and by G a special non distance distribution function in E given by

$$G(t) = \begin{cases} 1, & \text{if } t \leq 0 \\ 0, & \text{if } t > 0 \end{cases}$$

Definition 2.5. ([13]) The 5-tuple $(X, \mathcal{P}, \mathcal{Q}, *, \diamond)$ is said to be an intuitionistic Menger space if X is an arbitrary set, $*$ is a CTN, $\diamond$ is CTCN, $\mathcal{P}$ is a probabilistic distance and $\mathcal{Q}$ is a probabilistic non-distance on X satisfying the following conditions: for all $\alpha, \beta, \gamma \in X$ and $t, s \geq 0$

1. $\mathcal{P}_{(\alpha, \beta)}(t) + \mathcal{Q}_{(\alpha, \beta)}(t) \leq 1$,
2. $\mathcal{P}_{(\alpha, \beta)}(0) = 0$,
3. $\mathcal{P}_{(\alpha, \beta)}(t) = H(t)$ iff $\alpha = \beta$,
4. $\mathcal{P}_{(\alpha, \beta)}(t) = \mathcal{P}_{(\beta, \alpha)}(t)$
5. If $\mathcal{P}_{(\alpha, \beta)}(t) = 1$ and $\mathcal{P}_{(\beta, \gamma)}(s) = 1$, then $\mathcal{P}_{(\alpha, \gamma)}(t + s) = 1$,
6. $\mathcal{P}_{(\alpha, \gamma)}(t + s) \geq \mathcal{P}_{(\alpha, \beta)}(t) * \mathcal{P}_{(\beta, \gamma)}(s)$,
7. $\mathcal{Q}_{(\alpha, \beta)}(0) = 1$,
8. $\mathcal{Q}_{(\alpha, \beta)}(t) = G(t)$ iff $\alpha = \beta$,
9. $\mathcal{Q}_{(\alpha, \beta)}(t) = \mathcal{Q}_{(\beta, \alpha)}(t)$

10. If $Q_{(\alpha,\beta)}(t) = 0$ and $Q_{(\beta,\gamma)}(s) = 0$, then $Q_{(\alpha,\gamma)}(t+s) = 0$,
11. $Q_{(\alpha,\gamma)}(t+s) \leq Q_{(\alpha,\beta)}(t) \diamond Q_{(\beta,\gamma)}(s)$.

The function $\mathcal{P}_{(\alpha,\beta)}(t)$ and $Q_{(\alpha,\beta)}(t)$ denotes the degree of nearness and degree of non-nearness between α and β with respect to t , respectively.

Intuitionistic S_b -Menger Space:

Definition 2.6.[13] A six tuple $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ is an intuitionistic S_b -Menger Space ($IS_b - MS$) if $X \neq \phi$ is an arbitrary set, $*$ is a CTN, $\diamond$ is a CTCN, $k \geq 1$ is a real number, S is a probabilistic distance and ξ is a probabilistic non-distance on X satisfying the below properties for all : $\forall \alpha, \beta, \gamma, a \in X$ and $k, t, s > 0$.

- (i) $\mathcal{P}_{b(\alpha,\beta,\gamma)}(t) + Q_{b(\alpha,\beta,\gamma)}(t) \leq 1$,
- (ii) $\mathcal{P}_{b(\alpha,\beta,\gamma)}(t) > 0$;
- (iii) $\mathcal{P}_{b(\zeta\alpha,\beta,\gamma)}(t) = 1$ if and only if $\alpha = \beta = \gamma$;
- (iv) $\mathcal{P}_{b(\alpha,\beta,\gamma)}k(r+s+t) \geq \mathcal{P}_{b(\alpha,\alpha,a)}(r) * \mathcal{P}_{b(\beta,\beta,a)}(s) * \mathcal{P}_{b(\gamma,\gamma,a)}(t)$;
- (v) $Q_{b(\alpha,\beta,\gamma)}(t) > 0$;
- (vi) $Q_{b(\alpha,\beta,\gamma)}(t) = 0$ if and only if $\alpha = \beta = \gamma$;
- (vii) $Q_{b(\alpha,\beta,\gamma)}k(r+s+t) \leq Q_{b(\alpha,\alpha,a)}(r) \diamond Q_{b(\beta,\beta,a)}(s) \diamond Q_{b(\gamma,\gamma,a)}(t)$.

Definition 2.7.[13] Let $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ be an intuitionistic symmetric S_b -Menger space with $t * t \geq t$ and $(1-t) \diamond (1-t) \diamond (1-t) \leq (1-t)$ for all $0 < t < 1$. A sequence $\{\alpha_n\}$ in X is said to be convergent to $\alpha \in X$ if, for any $\varepsilon > 0$ and $k \in (0,1)$, there exists a positive integer $\mathbb{N}$ such that $\mathcal{P}_{b(\alpha_n, \alpha_n, \alpha)}(\varepsilon) > 1-k$ and $Q_{b(\alpha_n, \alpha_n, \alpha)}(\varepsilon) < k$ whenever $n \geq \mathbb{N}$.

Lemma 2.8.[13] Let $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ be an intuitionistic symmetric S_b -Menger space. If there exists a constant $k \in (0,1)$, and two elements $\alpha, \beta \in X$ such that for all $t > 0$

$$\mathcal{P}_{b(\alpha,\alpha,\beta)}(kt) \geq \mathcal{P}_{b(\alpha,\alpha,\beta)}(t) \text{ and } Q_{b(\alpha,\alpha,\beta)}(kt) \leq Q_{b(\alpha,\alpha,\beta)}(t).$$

Then $\alpha = \beta$.

Lemma 2.9.[13] Let $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ be an intuitionistic symmetric S_b -Menger space. with $t * t \geq t$ and $(1-t) \diamond (1-t) \diamond (1-t) \leq (1-t)$ for all $0 < t < 1$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be two sequences in X converges to α and β respectively. if $t \geq 0$ is a point of continuity of $\mathcal{P}_{b(\alpha,\alpha,\beta)}(\cdot)$ and $Q_{b(\alpha,\alpha,\beta)}(\cdot)$, then $\lim_{n \rightarrow \infty} \mathcal{P}_{b(\alpha_n, \alpha_n, \beta_n)}(t) = \mathcal{P}_{b(\alpha,\alpha,\beta)}(t)$ and $\lim_{n \rightarrow \infty} Q_{b(\alpha_n, \alpha_n, \beta_n)}(t) = Q_{b(\alpha,\alpha,\beta)}(t)$.

Definition 2.10 [13]. Let Ψ be the class of all non-decreasing mappings $\psi: X \times X \times X \rightarrow X$ and $\Psi: X \times X \times X \rightarrow X$, (where $X \in [0,1]$) such that

- (i) $\lim_{n \rightarrow \infty} \psi^n(s) = 1, \forall s \in (0,1]$,
- (ii) $\psi(s) > s, \forall s \in (0,1)$,
- (iii) $\psi(1) = 1$,
- (iv) $\lim_{n \rightarrow \infty} \eta^n(r) = 0, \forall r \in (0,1]$,

- (v) $\eta(r) < r, \forall r \in (0,1)$
 (vi) $\eta(0) = 0$.

3. Main Results:

Singh et al. [29, 30] introduced the notion of compatibility of type (E), A-compatibility of type (E) and S-compatibility of type (E), we define all these notions further in the structure of intuitionistic S_b -Menger space as follows:

Definition 3.1. Self-maps A and S on an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ are said to be compatible of type E , if $\lim_{n \rightarrow \infty} S^2 \alpha_n = \lim_{n \rightarrow \infty} SA \alpha_n = A\gamma$ and $\lim_{n \rightarrow \infty} A^2 \alpha_n = \lim_{n \rightarrow \infty} AS \alpha_n = S\gamma$, whenever $\{\alpha_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} A \alpha_n = \lim_{n \rightarrow \infty} S \alpha_n = \gamma$ for some $\gamma \in X$.

Definition 3.2. Self-maps A and S on an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ are said to be A -compatible of type (E), if $\lim_{n \rightarrow \infty} A^2 \alpha_n = \lim_{n \rightarrow \infty} AS \alpha_n = S\gamma$, whenever $\{\alpha_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} A \alpha_n = \lim_{n \rightarrow \infty} S \alpha_n = \gamma$ for some $\gamma \in X$. Pair A and S are said to be S -compatible of type (E), if $\lim_{n \rightarrow \infty} S^2 \alpha_n = \lim_{n \rightarrow \infty} SA \alpha_n = A\gamma$ whenever $\{\alpha_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} A \alpha_n = \lim_{n \rightarrow \infty} S \alpha_n = \gamma$ for some $\gamma \in X$.

Remark 3.3. It is also interesting to see that if A and S are compatible of type (E), then they are A -Compatible and S -Compatible of type (E), but the converse need not be true.

Definition 3.4. Self-maps A and S on $(IS_b-SMS)(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ are said to be sub-sequentially continuous, if there exist a sequence $\{\alpha_n\}$ such that $\lim_{n \rightarrow \infty} A \alpha_n = \lim_{n \rightarrow \infty} S \alpha_n = t$ for some $t \in X$ and $\lim_{n \rightarrow \infty} AS \alpha_n = At$ and $\lim_{n \rightarrow \infty} SA \alpha_n = St$.

Motivated by the Definition 2.6., we define the following in the setting of intuitionistic S_b -Menger space.

Definition 3.5. Self-maps A and S defined on an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ are said to be weakly subsequentially continuous, if there exists a sequence $\{\alpha_n\}$ such that $\lim_{n \rightarrow \infty} A \alpha_n = \lim_{n \rightarrow \infty} S \alpha_n = \gamma$ for some $\gamma \in X$ and $\lim_{n \rightarrow \infty} AS \alpha_n = A\gamma$ Or $\lim_{n \rightarrow \infty} SA \alpha_n = S\gamma$

Definition 3.6. Self-maps A and S defined on an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ are said to be S subsequentially continuous, if there exists a sequence $\{\alpha_n\}$ such that $\lim_{n \rightarrow \infty} A \alpha_n = \lim_{n \rightarrow \infty} S \alpha_n = \gamma$ for some $\gamma \in X$ and $\lim_{n \rightarrow \infty} SA \alpha_n = S\gamma$.

Definition 3.7. Self-maps A and S defined on an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ are said to be A -subsequentially continuous, if there exists a sequence $\{\alpha_n\}$ such that $\lim_{n \rightarrow \infty} A \alpha_n = \lim_{n \rightarrow \infty} S \alpha_n = \gamma$ for some $\gamma \in X$ and $\lim_{n \rightarrow \infty} AS \alpha_n = A\gamma$.

Remark 3.8. If the pair of mappings $\{A, S\}$ is A -subsequentially continuous (or S -subsequentially continuous) then it is (wsc) but never be subsequentially continuous.

Fixed Point Theorem:

Theorem 3.9 Let A, B, S and T be four self-mappings of an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ with CTN $*$ and CTCN $\diamond$ satisfying $t * t \geq t$ and $(1-t) \diamond (1-t) \diamond (1-t) \leq$

$(1 - t)$ for all $0 < t < 1$. Suppose $\psi, \eta \in \Psi$ and there exist a constant $k \in (0, 1)$ such that for all $\alpha, \beta \in X$ and $t > 0$ the following conditions are satisfied:

$$\mathcal{P}_{b(A\alpha, A\alpha, B\beta)}(kt) \geq \psi[\min\{\mathcal{P}_{b(S\alpha, S\alpha, T\beta)}(t), \mathcal{P}_{b(A\alpha, A\alpha, S\alpha)}(t), \mathcal{P}_{b(B\beta, B\beta, T\beta)}(t), \mathcal{P}_{b(S\alpha, S\alpha, B\beta)}(t), \mathcal{P}_{b(T\beta, T\beta, A\alpha)}(t)\}], \dots (3.9.1)$$

$$\mathcal{Q}_{b(A\alpha, A\alpha, B\beta)}(kt) \leq \eta[\max\{\mathcal{Q}_{b(S\alpha, S\alpha, T\beta)}(t), \mathcal{Q}_{b(A\alpha, A\alpha, S\alpha)}(t), \mathcal{Q}_{b(B\beta, B\beta, T\beta)}(t), \mathcal{Q}_{b(S\alpha, S\alpha, B\beta)}(t), \mathcal{Q}_{b(T\beta, T\beta, A\alpha)}(t)\}] \dots (3.9.2)$$

If the pairs $\{A, S\}$ and $\{B, T\}$ are weakly subsequential continuous and compatible of type (E) , then A, B, S and T have a unique common fixed point in X .

Proof. Since the pair $\{A, S\}$ is weakly subsequential continuous, we can assume that it is A – subsequential continuous and compatible of type (E) . There exists a sequence $\{\alpha_n\}$ in X such that $\lim_{n \rightarrow \infty} A\alpha_n = \lim_{n \rightarrow \infty} S\alpha_n = \gamma$ for some $\gamma \in X$ and $\lim_{n \rightarrow \infty} AS\alpha_n = A\gamma$. The compatibility of type (E) implies that $\lim_{n \rightarrow \infty} A^2\alpha_n = \lim_{n \rightarrow \infty} AS\alpha_n = S\gamma$ and $\lim_{n \rightarrow \infty} S^2\alpha_n = \lim_{n \rightarrow \infty} SA\alpha_n = A\gamma$. Therefore $A\gamma = S\gamma$, whereas in respect to the pair $\{B, T\}$, suppose that it is B – subsequential continuous. Then there exists a sequence $\{\beta_n\}$ in X such that $\lim_{n \rightarrow \infty} B\beta_n = \lim_{n \rightarrow \infty} T\beta_n = w$, for some $w \in X$ and $\lim_{n \rightarrow \infty} BT\beta_n = Bw$. The pair $\{B, T\}$ is compatible of type (E) , so $\lim_{n \rightarrow \infty} B^2\beta_n = \lim_{n \rightarrow \infty} BT\beta_n = Tw$. and $\lim_{n \rightarrow \infty} T^2\beta_n = \lim_{n \rightarrow \infty} TB\beta_n = Bw$ for some $w \in X$. This gives $Bw = Tw$. Hence γ is a coincidence point of the pair $\{A, S\}$ whereas w is a coincidence point of the pair $\{B, T\}$. Now we prove that $\gamma = w$. Choose a $t > 0$ satisfying (3.9.1) and (3.9.2). Without loss of generality, we can assume that t and kt are point of continuity of $\mathcal{P}_{b(\gamma, w, \dots)}(\cdot), \mathcal{P}_{b(\gamma, \gamma, \dots)}(\cdot), \mathcal{P}_{b(w, w, \dots)}(\cdot), \mathcal{Q}_{b(\gamma, w, \dots)}(\cdot), \mathcal{Q}_{b(\gamma, \gamma, \dots)}(\cdot), \mathcal{Q}_{b(w, w, \dots)}(\cdot), \mathcal{P}_{b(A\gamma, w, \dots)}(\cdot), \mathcal{P}_{b(S\gamma, w, \dots)}(\cdot), \mathcal{P}_{b(\gamma, B\gamma, \dots)}(\cdot), \mathcal{Q}_{b(\gamma, T\gamma, \dots)}(\cdot), \mathcal{Q}_{b(A\gamma, w, \dots)}(\cdot), \mathcal{Q}_{b(S\gamma, w, \dots)}, \mathcal{Q}_{b(\gamma, B\gamma, \dots)}(\cdot)$ and $\mathcal{Q}_{b(\gamma, T\gamma, \dots)}$. This is so because these functions are monotonic on $\mathbb{R}$ and hence have at most countable number of discontinuities in $(0, b)$ for any $b > 0$. So we may choose t sufficiently small that $0 < kt < t < b$ and both kt and t are points of continuity of all the functions mentioned above. By putting $\alpha = \alpha_n$ and $\beta = \beta_n$ in inequality (3.9.1), we have

$$\mathcal{P}_{b(A\alpha_n, A\alpha_n, B\beta_n)}(kt) \geq \psi[\min\{\mathcal{P}_{b(S\alpha_n, S\alpha_n, T\beta_n)}(t), \mathcal{P}_{b(A\alpha_n, A\alpha_n, S\alpha_n)}(t), \mathcal{P}_{b(B\beta_n, B\beta_n, T\beta_n)}(t), \mathcal{P}_{b(S\alpha_n, S\alpha_n, B\beta_n)}(t), \mathcal{P}_{b(T\beta_n, T\beta_n, A\alpha_n)}(t)\}]$$

Taking the limit as $n \rightarrow \infty$ and using Lemma(2.9), we get

$$\mathcal{P}_{b(\gamma, \gamma, w)}(kt) \geq \psi[\min\{\mathcal{P}_{b(\gamma, \gamma, w)}(t), \mathcal{P}_{b(\gamma, \gamma, \gamma)}(t), \mathcal{P}_{b(w, w, w)}(t), \mathcal{P}_{b(\gamma, \gamma, w)}(t), \mathcal{P}_{b(w, w, \gamma)}(t)\}].$$

so

$$\mathcal{P}_{b(\gamma, \gamma, w)}(kt) \geq \psi[\min\{\mathcal{P}_{b(\gamma, \gamma, w)}(t), 1, 1, \mathcal{P}_{b(\gamma, \gamma, w)}(t), \mathcal{P}_{b(w, w, \gamma)}(t)\}].$$

This gives, for all $t > 0$,

$$\mathcal{P}_{b(\gamma, \gamma, w)}(kt) \geq \psi[\mathcal{P}_{b(\gamma, \gamma, w)}(t)] \geq \mathcal{P}_{b(\gamma, \gamma, w)}(t). \dots (3.9.3)$$

Again, by putting $\alpha = \alpha_n$ and $\beta = \beta_n$ in inequality (3.9.2), we have

$$\begin{aligned} & Q_{b(A\alpha_n, A\alpha_n, B\beta_n)}(kt) \\ & \leq \eta[\max\{Q_{b(S\alpha_n, S\alpha_n, T\beta_n)}(t), Q_{b(A\alpha_n, A\alpha_n, S\alpha_n)}(t), Q_{b(B\beta_n, B\beta_n, T\beta_n)}(t), Q_{b(S\alpha_n, S\alpha_n, B\beta_n)}(t), Q_{b(T\beta_n, T\beta_n, A\alpha_n)}(t)\}] \end{aligned}$$

Taking the limit as $n \rightarrow \infty$ and using Lemma (2.9), we get

$$Q_{b(\gamma, \gamma, w)}(kt) \leq \eta[\max\{Q_{b(\gamma, \gamma, w)}(t), Q_{b(\gamma, \gamma, \gamma)}(t), Q_{b(w, w, w)}(t), Q_{b(\gamma, \gamma, w)}(t), Q_{b(w, w, \gamma)}(t)\}].$$

so, we have for all $t > 0$

$$Q_{b(\gamma, \gamma, w)}(kt) \leq \eta[\max\{Q_{b(\gamma, \gamma, w)}(t), 0, 0, Q_{b(\gamma, \gamma, w)}(t), Q_{b(w, w, \gamma)}(t)\}].$$

$$Q_{b(\gamma, \gamma, w)}(kt) \leq \eta[Q_{b(\gamma, \gamma, w)}(t)] \leq Q_{b(\gamma, \gamma, w)}(t). \dots (3.9.4)$$

for all $t > 0$,

By lemma (2.8), (3.9.3) and (3.9.4), we have $\gamma = w$. Now we prove that $A\gamma = \gamma$. By putting $\alpha = \gamma$ and $\beta = \beta_n$ in inequality (3.9.1), we get

$$\begin{aligned} & P_{b(A\gamma, A\gamma, B\beta_n)}(kt) \\ & \geq \psi[\min\{P_{b(S\gamma, S\gamma, T\beta_n)}(t), P_{b(A\gamma, A\gamma, S\gamma)}(t), P_{b(B\beta_n, B\beta_n, T\beta_n)}(t), P_{b(S\gamma, S\gamma, B\beta_n)}(t), P_{b(T\beta_n, T\beta_n, A\gamma)}(t)\}] \end{aligned}$$

Letting $n \rightarrow \infty$ and using Lemma (2.8), we obtain

$$\begin{aligned} & P_{b(A\gamma, A\gamma, w)}(kt) \\ & \geq \psi[\min\{P_{b(S\gamma, S\gamma, w)}(t), P_{b(A\gamma, A\gamma, S\gamma)}(t), P_{b(w, w, w)}(t), P_{b(S\gamma, S\gamma, w)}(t), P_{b(w, w, A\gamma)}(t)\}] \end{aligned}$$

This gives

$$P_{b(A\gamma, A\gamma, w)}(kt) \geq \psi[\min\{P_{b(S\gamma, S\gamma, w)}(t), 1, 1, P_{b(S\gamma, S\gamma, w)}(t), P_{b(w, w, A\gamma)}(t)\}]. \text{ But } A\gamma = S\gamma. \text{ Thus, for all } t > 0,$$

$$P_{b(A\gamma, A\gamma, w)}(kt) \geq \psi[P_{b(A\gamma, A\gamma, w)}(t)] \geq P_{b(A\gamma, A\gamma, w)}(t). \dots (3.9.5)$$

Again, by putting $\alpha = \gamma$ and $\beta = \beta_n$ in inequality (3.9.2), we get

$$\begin{aligned} & Q_{b(A\gamma, A\gamma, B\beta_n)}(kt) \\ & \leq \eta[\max\{Q_{b(S\gamma, S\gamma, T\beta_n)}(t), Q_{b(A\gamma, A\gamma, S\gamma)}(t), Q_{b(B\beta_n, B\beta_n, T\beta_n)}(t), Q_{b(S\gamma, S\gamma, B\beta_n)}(t), Q_{b(T\beta_n, T\beta_n, A\gamma)}(t)\}] \end{aligned}$$

Taking the limit $n \rightarrow \infty$ and using Lemma (2.8), we obtain

$$Q_{b(A\gamma, A\gamma, w)}(kt) \leq \eta[\max\{Q_{b(S\gamma, S\gamma, w)}(t), Q_{b(A\gamma, A\gamma, S\gamma)}(t), Q_{b(w, w, w)}(t), Q_{b(S\gamma, S\gamma, w)}(t), Q_{b(w, w, A\gamma)}(t)\}]$$

This gives

$$Q_{b(A\gamma, A\gamma, w)}(kt) \leq \eta[\max\{Q_{b(S\gamma, S\gamma, w)}(t), 0, 0, Q_{b(S\gamma, S\gamma, w)}(t), Q_{b(w, w, A\gamma)}(t)\}] \text{ and so for all } t > 0,$$

$$Q_{b(A\gamma, A\gamma, w)}(kt) \leq \eta[Q_{b(A\gamma, A\gamma, w)}(t)] \leq Q_{b(A\gamma, A\gamma, w)}(t) \dots (3.9.6)$$

By lemma (2.8), (3.9.5) and (3.9.6) we get, $A\gamma = w$. Since $A\gamma = S\gamma$, we have $A\gamma = S\gamma = w = \gamma$.

Now we prove that $B\gamma = \gamma$.

By putting $\alpha = \alpha_n$ and $\beta = \gamma$ in inequality (3.9.1), we get

$$\mathcal{P}_{b(A\alpha_n, A\alpha_n, B\gamma)}(kt) \geq \psi[\min\{\mathcal{P}_{b(S\alpha_n, S\alpha_n, T\gamma)}(t), \mathcal{P}_{b(A\alpha_n, A\alpha_n, S\alpha_n)}(t), \mathcal{P}_{b(B\gamma, B\gamma, T\gamma)}(t), \mathcal{P}_{b(S\alpha_n, S\alpha_n, B\gamma)}(t), \mathcal{P}_{b(T\gamma, T\gamma, A\alpha_n)}(t)\}]$$

Taking the limit as $n \rightarrow \infty$ and using Lemma 2.8, we obtain

$$\mathcal{P}_{b(\gamma, \gamma, B\gamma)}(kt) \geq \psi[\min\{\mathcal{P}_{b(\gamma, \gamma, T\gamma)}(t), \mathcal{P}_{b(\gamma, \gamma, \gamma)}(t), \mathcal{P}_{b(B\gamma, B\gamma, T\gamma)}(t), \mathcal{P}_{b(\gamma, \gamma, B\gamma)}(t), \mathcal{P}_{b(T\gamma, T\gamma, \gamma)}(t)\}]$$

$$\mathcal{P}_{b(\gamma, \gamma, B\gamma)}(kt) \geq \psi[\min\{\mathcal{P}_{b(\gamma, \gamma, T\gamma)}(t), 1, 1, \mathcal{P}_{b(\gamma, \gamma, B\gamma)}(t), \mathcal{P}_{b(T\gamma, T\gamma, \gamma)}(t)\}]$$

Since $\gamma = w$ and $Bw = Tw$, then $B\gamma = T\gamma$. Thus, for all $t > 0$,

$$\mathcal{P}_{b(\gamma, \gamma, B\gamma)}(kt) \geq \psi[\mathcal{P}_{b(\gamma, \gamma, B\gamma)}(t)] \geq \mathcal{P}_{b(\gamma, \gamma, B\gamma)}(t) \dots (3.9.7)$$

Again, by putting $\alpha = \alpha_n$ and $\beta = \gamma$ in inequality (3.9.2), we get

$$\mathcal{Q}_{b(A\alpha_n, A\alpha_n, B\gamma)}(kt) \leq \eta[\max\{\mathcal{Q}_{b(S\alpha_n, S\alpha_n, T\gamma)}(t), \mathcal{Q}_{b(A\alpha_n, A\alpha_n, S\alpha_n)}(t), \mathcal{Q}_{b(B\gamma, B\gamma, T\gamma)}(t), \mathcal{Q}_{b(S\alpha_n, S\alpha_n, B\gamma)}(t), \mathcal{Q}_{b(T\gamma, T\gamma, A\alpha_n)}(t)\}]$$

Taking the limit as $n \rightarrow \infty$ and using Lemma 2.8, we obtain

$$\mathcal{Q}_{b(\gamma, \gamma, B\gamma)}(kt) \leq \eta[\max\{\mathcal{Q}_{b(\gamma, \gamma, T\gamma)}(t), \mathcal{Q}_{b(\gamma, \gamma, \gamma)}(t), \mathcal{Q}_{b(B\gamma, B\gamma, T\gamma)}(t), \mathcal{Q}_{b(\gamma, \gamma, B\gamma)}(t), \mathcal{Q}_{b(T\gamma, T\gamma, \gamma)}(t)\}]$$

which gives

$$\mathcal{Q}_{b(\gamma, \gamma, B\gamma)}(kt) \leq \eta[\max\{\mathcal{Q}_{b(\gamma, \gamma, T\gamma)}(t), 0, 0, \mathcal{Q}_{b(\gamma, \gamma, B\gamma)}(t), \mathcal{Q}_{b(T\gamma, T\gamma, \gamma)}(t)\}]$$

and so for all $t > 0$,

$$\mathcal{Q}_{b(\gamma, \gamma, B\gamma)}(kt) \leq \eta[\mathcal{Q}_{b(\gamma, \gamma, B\gamma)}(t)] \leq \mathcal{Q}_{b(\gamma, \gamma, B\gamma)}(t) \dots \dots (3.9.8)$$

By lemma (2.8), (3.9.7) and (3.9.8), we obtain, $B\gamma = \gamma$. Since, $B\gamma = T\gamma$, we have $B\gamma = T\gamma = \gamma$. So in all $\gamma = A\gamma = B\gamma = S\gamma = T\gamma$, that is, γ is a common fixed point of A, B, S and T .

To prove uniqueness, let $\gamma^* \in X$ be such that $\gamma^* = A\gamma^* = B\gamma^* = S\gamma^* = T\gamma^*$.

By putting $\alpha = \gamma$ and $\beta = \gamma^*$ in inequality (3.9.1), we get

$$\mathcal{P}_{b(A\gamma, A\gamma, B\gamma^*)}(kt) \geq \psi[\min\{\mathcal{P}_{b(S\gamma, S\gamma, T\gamma^*)}(t), \mathcal{P}_{b(A\gamma, A\gamma, S\gamma)}(t), \mathcal{P}_{b(B\gamma^*, B\gamma^*, T\gamma^*)}(t), \mathcal{P}_{b(S\gamma, S\gamma, B\gamma^*)}(t), \mathcal{P}_{b(T\gamma^*, T\gamma^*, A\gamma)}(t)\}],$$

That is

$$\mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(kt) \geq \psi[\min\{\mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(t), \mathcal{P}_{b(\gamma, \gamma, \gamma)}(t), \mathcal{P}_{b(\gamma^*, \gamma^*, \gamma^*)}(t), \mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(t), \mathcal{P}_{b(\gamma^*, \gamma^*, \gamma)}(t)\}]$$

So,

$$\mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(kt) \geq \psi[\min\{\mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(t), 1, 1, \mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(t), \mathcal{P}_{b(\gamma^*, \gamma^*, \gamma)}(t)\}] \text{ thus, for all } t > 0,$$

$$\mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(kt) \geq \psi[\mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(t)] \geq \mathcal{P}_{b(\gamma, \gamma, \gamma^*)}(t) \dots (3.9.9)$$

By putting $\alpha = \gamma$ and $\beta = \gamma^*$ in inequality (3.9.2), we get

$$\mathcal{Q}_{b(A\gamma, A\gamma, B\gamma^*)}(kt) \leq \eta[\max\{\mathcal{Q}_{b(S\gamma, S\gamma, T\gamma^*)}(t), \mathcal{Q}_{b(A\gamma, A\gamma, S\gamma)}(t), \mathcal{Q}_{b(B\gamma^*, B\gamma^*, T\gamma^*)}(t), \mathcal{Q}_{b(S\gamma, S\gamma, B\gamma^*)}(t), \mathcal{Q}_{b(T\gamma^*, T\gamma^*, A\gamma)}(t)\}]$$

That is

$$Q_{b(\gamma, \gamma, \gamma^*)}(kt) \leq \eta[\max\{Q_{b(\gamma, \gamma, \gamma^*)}(t), Q_{b(\gamma, \gamma, \gamma)}(t), Q_{b(\gamma^*, \gamma^*, \gamma^*)}(t), Q_{b(\gamma, \gamma, \gamma^*)}(t), Q_{b(\gamma^*, \gamma^*, \gamma)}(t)\}]$$

So,

$$Q_{b(\gamma, \gamma, \gamma^*)}(kt) \leq \eta[\max\{Q_{b(\gamma, \gamma, \gamma^*)}(t), 0, 0, Q_{b(\gamma, \gamma, \gamma^*)}(t), Q_{b(\gamma^*, \gamma^*, \gamma)}(t)\}]$$

Thus, for all $t > 0$,

$$Q_{b(\gamma, \gamma, \gamma^*)}(kt) \leq \eta[Q_{b(\gamma, \gamma, \gamma^*)}(t)] \leq Q_{b(\gamma, \gamma, \gamma^*)}(t).$$

By Lemma 2.8, (3.9.9) and (3.9.10) we get, $\gamma = \gamma^*$.

If we put $A = B$ in Theorem 3.9 we have the following corollary for three mappings:

Corollary 3.10. Let A, S and T be three self-mappings of an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ with CTN $*$ and CTCN $\diamond$ satisfying $t * t \geq t$ and $(1-t) \diamond (1-t) \diamond (1-t) \leq (1-t)$ for all $0 < t < 1$. Suppose $\psi, \eta \in \Psi$ and there exist a constant $k \in (0, 1)$ such that for all $\alpha, \beta \in X$ and $t > 0$ the following conditions are satisfied:

$$\begin{aligned} &\mathcal{P}_{b(A\alpha, A\alpha, A\beta)}(kt) \geq \\ &\psi[\min\{\mathcal{P}_{b(S\alpha, S\alpha, T\beta)}(t), \mathcal{P}_{b(A\alpha, A\alpha, S\alpha)}(t), \mathcal{P}_{b(A\beta, A\beta, T\beta)}(t), \mathcal{P}_{b(S\alpha, S\alpha, A\beta)}(t), \mathcal{P}_{b(T\beta, T\beta, A\alpha)}(t)\}], \text{ and} \\ &Q_{b(A\alpha, A\alpha, A\beta)}(kt) \\ &\leq \eta[\max\{Q_{b(S\alpha, S\alpha, T\beta)}(t), Q_{b(A\alpha, A\alpha, S\alpha)}(t), Q_{b(A\beta, A\beta, T\beta)}(t), Q_{b(S\alpha, S\alpha, A\beta)}(t), Q_{b(T\beta, T\beta, A\alpha)}(t)\}] \end{aligned}$$

If the pairs $\{A, S\}$ and $\{A, T\}$ are weakly subsequential continuous and compatible of type (E) , then A, S and T have a unique common fixed point in X .

Alternatively, if we set $S = T$ in theorem (3.1), we will have the following corollary for three self-mappings.

Corollary 3.11. Let A, B and S be three self-mappings of an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ with CTN $*$ and CTCN $\diamond$ satisfying $t * t \geq t$ and $(1-t) \diamond (1-t) \diamond (1-t) \leq (1-t)$ for all $0 < t < 1$. Suppose $\psi, \eta \in \Psi$ and there exist a constant $k \in (0, 1)$ such that for all $\alpha, \beta \in X$ and $t > 0$ the following conditions are satisfied:

$$\begin{aligned} &\mathcal{P}_{b(A\alpha, A\alpha, B\beta)}(kt) \geq \\ &\psi[\min\{\mathcal{P}_{b(S\alpha, S\alpha, S\beta)}(t), \mathcal{P}_{b(A\alpha, A\alpha, S\alpha)}(t), \mathcal{P}_{b(B\beta, B\beta, S\beta)}(t), \mathcal{P}_{b(S\alpha, S\alpha, B\beta)}(t), \mathcal{P}_{b(S\beta, S\beta, A\alpha)}(t)\}], \text{ and} \\ &Q_{b(A\alpha, A\alpha, B\beta)}(kt) \\ &\leq \eta[\max\{Q_{b(S\alpha, S\alpha, S\beta)}(t), Q_{b(A\alpha, A\alpha, S\alpha)}(t), Q_{b(B\beta, B\beta, S\beta)}(t), Q_{b(S\alpha, S\alpha, B\beta)}(t), Q_{b(S\beta, S\beta, A\alpha)}(t)\}] \end{aligned}$$

If the pairs $\{A, S\}$ and $\{B, S\}$ are weakly subsequential continuous and compatible of type (E) , then A, B and S have a unique common fixed point in X .

Alternatively, if we set $S = T$ in corollary (3.3), we will have the following corollary for two self-mappings.

Corollary 3.12. Let A, S be two self-mappings of an intuitionistic symmetric S_b -Menger space $(X, \mathcal{P}_b, Q_b, *, \diamond, k)$ with CTN $*$ and CTCN $\diamond$ satisfying $(1-t) \diamond (1-t) \diamond (1-t) \leq (1-t)$ for all $0 < t < 1$. Suppose $\psi, \eta \in \Psi$ and there exist a constant $k \in (0, 1)$ such that for all $\alpha, \beta \in X$ and $t > 0$ the following conditions are satisfied:

$$\begin{aligned} & \mathcal{P}_{b(A\alpha, A\alpha, A\beta)}(kt) \geq \\ & \psi[\min\{\mathcal{P}_{b(S\alpha, S\alpha, S\beta)}(t), \mathcal{P}_{b(A\alpha, A\alpha, S\alpha)}(t), \mathcal{P}_{b(A\beta, A\beta, S\beta)}(t), \mathcal{P}_{b(S\alpha, S\alpha, A\beta)}(t), \mathcal{P}_{b(S\beta, S\beta, A\alpha)}(t)\}] \text{ and} \\ & \mathcal{Q}_{b(A\alpha, A\alpha, A\beta)}(kt) \\ & \leq \eta[\max\{\mathcal{Q}_{b(S\alpha, S\alpha, S\beta)}(t), \mathcal{Q}_{b(A\alpha, A\alpha, S\alpha)}(t), \mathcal{Q}_{b(A\beta, A\beta, S\beta)}(t), \mathcal{Q}_{b(S\alpha, S\alpha, A\beta)}(t), \mathcal{Q}_{b(S\beta, S\beta, A\alpha)}(t)\}] \end{aligned}$$

If the pairs $\{A, S\}$ are weakly subsequential continuous and compatible of type (E) , then A and S have a unique common fixed point in X .

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